New Thermostats for Multiscaling Polymer Simulation

SUNITA ANDREEA MOGA[1], NICOLAE GOGA[1,2], ANTON HADAR[1]*

[1] Politehnica University of Bucharest, 313 Splaiul Independentei, Bucharest, Romania

[2] Groningen Biomolecular Sciences and Biotechnology Institute University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

In this article we present our work on the combination of two thermostats, motivated by our research on multiscaling molecular dynamics simulations. Molecular dynamics are used for studying the properties of polymers. Two schemes - global Berendsen and stochastic dynamics - are combined. A leap-frog algorithm was used for the calculation of $k_{thermal}$*. Simulation results on temperature, thermal rate, and diffusion for coarse-grained (MARTINI) water are discussed. Using these thermostats we observe no deviation from the reference temperature and show that the measured thermal relaxation rates agree well with theoretical predictions.*

Keywords: molecular dynamics, polymers, thermostats

The recent advances in the field of molecular dynamics (MD) multiscaling have prompted for the development of new thermostatting schemes. Molecular dynamics are used for studying the properties of polymers [1 - 4].

The concepts of MD multiscaling, coarse-, and fine-graining are tightly connected to the dual thermostat scheme we develop here - indeed, it was our research on MD multiscaling that motivated this work. We begin, therefore, with a brief overview of these concepts. In the field of molecular dynamics, traditional all-atom - or fine-grained (FG) - MD simulations are capable of providing most detailed information, down to the atomic level. Unfortunately, the limited time and system sizes that can be addressed prevents detailed simulation of collective phenomena in many but the most trivial systems. A way to overcome these limitations, extending the molecular modeling and bridging it with experimental techniques, is to use coarse-graining: i.e., to represent a system by a reduced number of degrees of freedom relative to the all-atom description. Due to the fewer degrees of freedom and the elimination of fine interaction details the simulation of a coarse-grained (CG) system is less resource-intensive than the same system in an all-atom representation. As a result, an increase of orders of magnitude in the simulated time and length scales can be achieved. The drawback of this approach, however, is that important detail can be lost in the approximations introduced by the coarse-graining process. A novel solution for this problem is to use multiscaling simulations. The theoretical background of multiscaling algorithms we have researched can be found in [5]. The need for new types of thermostats showed up in our attempts to perform constant-λ multiscale simulations. The constant - λ approach [6] consists of a linear interpolation of the potential at the fine-grained level V^{FG} with the one from the coarse-grained level V^{CG}. The total potential reads $V = \lambda V^{FG} + (1 - \lambda) V^{CG}$. In this particular hybrid method the system is thus described at two levels of resolution simultaneously, ideally suited for Hamiltonian exchange simulations in which concurrent simulations with different λ - values are coupled via the replica exchange method. To maintain a proper Boltzmann distribution, FG subsystem must be maintained at a λ - scaled temperature. This can be accomplished using two different thermostats for each of the CG and FG subsystems. Such a control of the FG reference temperature yields proper results for higher values of λ; however, for smaller values the CG subsystem looses energy (in the form of heat, i.e. kinetic energy) to the FG subsystem, which is at a (much) lower temperature.

This is to be expected since these two subsystems are coupled. For such situations, global thermostat algorithms lead to a so-called ice-cube effect, in which the energy of high-frequency modes is drained into low-frequency modes, particularly into zero-frequency motions such as overall translation. Our attempts at solving this problem included the use a Langevin stochastic thermostat since such a thermostat is coupled to every degree of freedom separately; we found, however, that this destroyed the dynamic properties of the system, making the essence of the multiscale idea (increased sampling of phase space) pointless. Another approach involved the use of Dissipative Particle Dynamics (DPD) thermostats: we developed thermostatting schemes that, while more efficient than the ones from the literature, still displayed the shortcomings of the stochastic thermostat - even though the dynamic properties were to some extent better preserved. From these results we directed our efforts at combining global thermostats with stochastic ones. We remind that the techniques presented in this article can be applied for studying the properties of polymers, for their design and building new materials based on polymers. In this paper we will concentrate in presenting the theoretical background, properties, and simulation results of a mixed thermostat obtained by combination of global Berendsen with a stochastic type.

Theoretical part

In this section, we will discuss the theoretical approach of the combination of two thermostats. The first subsection is an overview of the global Berendsen thermostat. The second subsection gives the theory for the stochastic thermostat. Then a combination of the two thermostats into a combined one is presented. The last section presents an overview for the thermal rate theory.

Global Berendsen

In what follows we present an outline of the relevant theory for global Berendsen thermostat. For an exhaustive

---

* *email: antonhadar@yahoo.com*

presentation we refer to [7 - 8]. Let us summarize bellow this theory.

A coupling between a molecular system with temperature T and a bath [7-8] with temperature T_{ref} can be realized by inserting extra terms - friction and stochastic - in the equations of motions, which will result in a Langevin equation

$$m_i \nu_i^2 = F_i - m_i \gamma_i \nu_i + R(t)\ , \tag{1}$$

where:

- R_i is a stochastic Gaussian variable with null mean and with intensity

$$\langle R_i(t) R_j(t+\tau) = 2 m_i \gamma_i k_B T_{ref} \delta(\tau) \delta_{ij}. \tag{2}$$

- F_i - the systematic force.

The strength of the coupling to the bath is determined by the γ_i constants. The correspondent physical phenomenon for equation (1) is the one of frequent collisions with light particles that forms an ideal gas at temperature T_{ref}.

Through the Langevin equation the system is locally subject to random noise and couples globally to a heat bath. In order to impose global coupling with minimal local disturbance we should modify (1) so that only the global coupling remains.

We will analyze how the system temperature, *T*, behaves under the influence of stochastic coupling.

For an easier analytical computation we chose the friction constant to be the same for all particles: $\gamma_i = \gamma$. This is a matter of choice; different classes of degrees of freedom can be coupled to the bath with different friction constants. From the derivation of the kinetic energy E_k we can see the time dependence of *T*:

$$\frac{dE_k}{dt} = \lim_{\Delta t \to 0} \frac{\sum_{i=1}^{3N} \frac{1}{2} m_i v_i^2(t+\Delta t) - \sum_{i=1}^{3N} \frac{1}{2} m_i v_i^2(t)}{\Delta t}\ , \tag{3}$$

- where N is the number of particles, and

$$\Delta v_i = v_i(t+\Delta t) - v_i(t) = \frac{1}{m_i} \int_t^{t+\Delta t} [F_i(t') - m_i \gamma v_i(t') + R_i(t')] dt \tag{4}$$

Using the relation (2)

$$\sum_{i=1}^{3N} \int_t^{t+\Delta t} dt' \int_t^{t+\Delta t} dt'' R_i(t') R_i(t'') = 6Nm\gamma k_B T_{ref} \Delta t, \tag{5}$$

- and the fact that $R_i(t')$ is uncorrelated with $\nu_i(t)$ and $R_i(t)$ for $t' > t$, we obtain

$$\frac{dE_k}{dt} = \sum_{i=1}^{3N} v_i F_i + 2\gamma\left(\frac{3N}{2} k_B T_{ref} - E_k\right). \tag{6}$$

Let's us look to the different terms from equation (6). The first term corresponds to the time derivative of the potential energy with the sign minus; the second term is an additional term describing the global coupling to the heat bath. In terms of temperature this extra term looks like the following:

$$\left(\frac{dT}{dt}\right)_{bath} = 2\gamma(T_{ref} - T). \tag{7}$$

We can remark that the time constant τ_T equals $(2\gamma)^{-1}$. Returning to (1), it can be noted that the global additional temperature coupling (7) is accomplished by the equations

$$m_i v'_i = F_i + m_i \gamma\left(\frac{T_{ref}}{T} - 1\right) v_i \tag{8}$$

without adding local stochastic terms, because from (8) it follows that:

$$\frac{dE_k}{dt} = \sum m_i v_i v'_i = \sum_{i=1}^{3N} v_i F_i + 3N\gamma k_B (T_{ref} - T), \tag{9}$$

which is equivalent to (6). In this way we come to (8) as our new equation of motion. This represents a proportional scaling of the velocities per time step in the algorithm from ν to $\lambda\nu$ with (to first order)

$$\lambda = 1 + \frac{\Delta t}{2\tau_T}\left(\frac{T_{ref}}{T} - 1\right) v_i. \tag{10}$$

The change of temperature per step can also made exactly equal to $\frac{(T_{ref} - T)\Delta t}{\tau_T}$, yielding

(11)

$$\lambda^2 - 1 = \frac{\Delta t}{k_{Ber}}\left(\frac{T_{ref}}{T} - 1\right), \tag{12}$$

where $k_{Ber} = \frac{0.5 * 3Nk_B}{C_V \tau_T}$

and $C_V = N^* c_v$ (13)

is the heat capacity of the system.

Combining equations (11) and (12) we reach one of the terms used in the calculation of the rate constant $(k_{thermal})$ of the combined thermostat.

$$k_{Ber} = \frac{1.5 k_B}{c_V \tau_T} \tag{14}$$

Stochastic Dynamics

For development of the stochastic dynamics thermostat let us consider every degree of freedom separately [9]. Let us also assume $v(t - \frac{1}{2}\Delta t)$, $x(t)$ and $F(t) = F$ as the known velocity, coordinate and force components of that degree of freedom at the beginning of the time step. The impulsive Langevin extension of the leap-frog algorithm then reads as follows:

For all degrees of freedom do:

1. $v = v(t - \frac{1}{2}\Delta t) + (F/m)\Delta t$
2. $\Delta v = -fv + \sqrt{f(2-f)(k_B T_{ref}/m)}\,\xi$
3. $x(t + \Delta t) = x(t) + (v + \frac{1}{2}\Delta v)\Delta t$
4. $v(t + \frac{1}{2}\Delta t) = v + \Delta v$

Here step 1 is the usual MD velocity-update of the leap-frog scheme, step 2 is the impulsive application of friction (reducing the velocity by a fraction $f: 0 \le f \le 1$ and noise (ξ is a random sample from a normal distribution). Step 3 updates the coordinates, taking into account that $\Delta\nu$ is applied only between $t + (1/2)\Delta t$ and $t + \Delta t$: in fact, step 3 can be considered as two half steps (fig. 1):

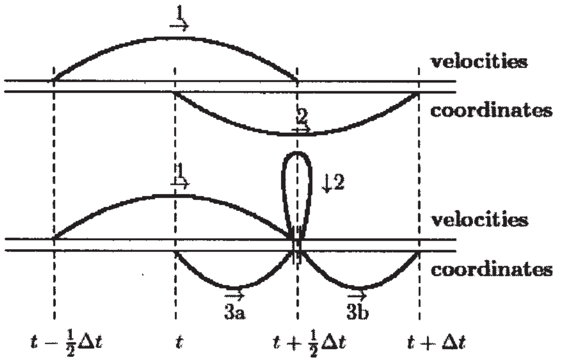

Fig. 1: (top) Traditional leap-frog scheme, (bottom) Leap-frog scheme with impulsive phase

3a. $x(t+\frac{1}{2}\Delta t)=x(t)+v\frac{1}{2}\Delta t,$

3b. $x(t+\Delta t)=x(t+\frac{1}{2}\Delta t)+(v+\Delta v)\frac{1}{2}\Delta t$.

Step 4 assigns the modified velocity to the velocity at the end of the time step.

The variance of the noise term is chosen such that the variance of the velocity

$$\left\langle (v+\Delta v)^2\right\rangle=(1-f)^2\left\langle v^2\right\rangle+f(2-f)\frac{k_BT_{ref}}{m} \tag{15}$$

tends to the stationary value of k_BT_{ref}/m. This is easily seen from (15) by substituting k_BT_{ref}/m for each of the mean squared velocities.

Note that the limiting case $f = 1$ completely removes the velocity and replaces it by a sample from a Maxwellian distribution. Thus, if the impulsive friction and noise is applied with $f = 1$ and with a probability $\Gamma\Delta t$ per particle per step, the Andersen thermostat is recovered. A smoothed Andersen thermostat with the same average velocity reduction factor will be obtained by applying the impulsive friction and noise with $f = \Gamma\Delta t$ every step to every degree of freedom. The algorithm is expected to be robust, in the sense that the impulsive term is exact, independently of the used time step. There is a lot of freedom of choice in the way the impulsive term is applied: the damping factor f may be applied to a random selection of particles and may differ for different particles. The application to velocity differences as in DPD is straightforward. In three dimensions the procedure is exactly the same as in one dimension. When the friction is isotropic, the algorithm is applied isotropically to a vector υ. The impulsive change is made with the same factor f on all three components of υ, but the random term is chosen differently with three independent random numbers ξ_x, ξ_y, ξ_z: For all particles do:

1. $v=v(t-\frac{1}{2}\Delta t)+(F/m)\Delta t$

2. $\Delta v=-fv+\sqrt{f(2-f)(k_BT_{ref}/m)}\begin{pmatrix}\xi_x\\ \xi_y\\ \xi_z\end{pmatrix}$

3. $r(t+\Delta t)=r(t)+(v+\frac{1}{2}\Delta v)\Delta t$

4. $v(t+\frac{1}{2}\Delta t)=v+\Delta v$

We see that the new velocity does not have the same direction as the old velocity. Obviously the total momentum is not conserved and neither is the total energy conserved. However, the average kinetic energy and hence the temperature will be stable: they tend toward the values determined by the reference temperature. Now, consider one dimension as treated above. The temperature is given by:

$$T=\frac{m}{k_B}\left\langle v^2\right\rangle. \tag{16}$$

The energy change ΔE_1 resulting from a single application of friction and noise to one degree of freedom is

$$\Delta E_1=\frac{1}{2}m(v+\Delta v)^2-\frac{1}{2}mv^2. \tag{17}$$

Using (15) and (16) this rewrites to

$$\Delta E_1=\frac{1}{2}f(2-f)k_B(T_{ref}-T); \tag{18}$$

the total energy change per time step Δt is the sum over all one-dimensional frictional events that occur per time step:

$$\Delta E_{tot}=\frac{1}{2}k_B\sum f(2-f)(T_{ref}-T). \tag{19}$$

The energy change is initially supplied to the kinetic energy of the system, thus changing the temperature. However, when the rate of change is small, the energy change will be distributed over kinetic and potential energy. The temperature change is then determined by the total heat capacity C_V of the system:

$$\Delta T=\frac{\Delta E_{tot}}{C_V}, \tag{20}$$

yielding a differential equation for the time-dependence of the temperature:

$$\frac{dT}{dt}=\frac{1}{2}k_B\frac{\sum f(2-f)}{C_V\Delta t}(T_{ref}-T). \tag{21}$$

In the case of a three-dimensional application to N particles (21) has the form

$$\frac{dT}{dt}=\frac{3k_B}{2c_V}\frac{f(2-f)}{\Delta t}(T_{ref}-T), \tag{22}$$

$$\text{where } c_V = C_V/N \tag{23}$$

is the specific heat per particle.

Equation (22) shows that any deviation from the reference temperature will decay to zero according to a first-order kinetic process

$$\frac{dT}{dt}=k_{SD}(T_{ref}-T) \tag{24}$$

with rate constant

$$k_{SD}=\frac{3k_B}{2c_V}\frac{f(2-f)}{\Delta t}. \tag{25}$$

Alternatively the decay can be characterized by a time constant $\tau_T = 1 / k_{SD}$. Note that for an ideal gas $c_V=\frac{3}{2}k_B$ reducing the left fraction in (25) to 1; for atomic fluids this fraction is usually 2 to 3 times smaller.

The thermal rate constant can be expressed in the friction constant γ :

$$k_{SD} = \frac{3k_B}{2c_V}\frac{[1 - exp(-2\gamma\Delta t)]}{\Delta t} \approx \frac{3k_B}{2c_V}2\gamma \qquad (26)$$

the latter value being a good approximation for small $\gamma\Delta t$. This equation provides the second term of the rate constant for the combined thermostat.

Combining two thermostats

When we combine the global Berendsen thermostat and the SD thermostat we will have an impulsive Langevin extension of the leap-frog algorithm that reads as follows:

For all particles do:

1. $v = \lambda[v(t - \frac{1}{2}\Delta t) + (F/_m)\Delta t]$, with λ computed according to formula (10)
2. $\Delta v = -fv + \sqrt{f(2 - f)(k_B T_{ref}/_m)}\begin{pmatrix}\xi_x\\ \xi_y\\ \xi_z\end{pmatrix}$
3. $r(t + \Delta t) = r(t) + (v + \frac{1}{2}\Delta v)\Delta t$
4. $v(t + \frac{1}{2}\Delta t) = v + \Delta v$

The main difference, as compared with the stochastic thermostat, is that on step 1 we have the λ of the global Berendsen thermostat computed according to formula (10).

Thermal rate theory

When we combine the two thermostats, the rate constant $k_{thermal}$ is given by:

$$k_{thermal} = k_{Ber} + k_{SD} \qquad (27)$$

According to formula (12) and (26), the rate constant $k_{thermal}$,is expressed as:

$$k_{thermal} = \frac{0.5nk_B}{C_V\tau_T} + \frac{3k_B}{2c_V}2\gamma \qquad (28)$$

where n is the total number of degrees of freedom of the total system and C_V, τ_T, k_B, c_V have the same meanings as in formulae (12) and (26).

Experimental part

Simulation details

The algorithms presented in the previous sections were implemented in the GROMACS program package [10] version 4.5.5, using parallelization based on domain decomposition.On this section, we will present the simulation results of the new algorithms on MARTINI course grained water [11] which was used to test performance of the investigated coupling scheme. In future we plan to apply the presented algorithms to more complicated molecular systems, namely to polymers for studying their properties.

All simulations for MARTINI water were performed in a periodic cubic box with dimensions longer than twice the cut-off distance. A cut-off distance was set at 1.2 nm. To minimize cut-off artifacts the potential was modified by a shift function to be zero at the cutoff, the force decaying smoothly to zero from 0.9 nm to the cutoff [12]. In all simulations a time step of 2 fs was used. The reference temperature was set to 320K.The MARTINI water systems consisted of 3200 particles with mass 72 u in a cubic box of (7.28856 nm)^3. The initial velocities of the particles were obtained from a Maxwell-Boltzmann distribution corresponding to the chosen initial reference temperature. Simulations where either 2 ns or 1 ns long, for diffusion and thermal rate calculations, respectively.

For the computation of the diffusion constant, we used the mean square displacement (MSD) and applied the g_{msd} function of Gromacs. This function computes the mean square displacement of atoms from a set of initial positions.This provides an easy way to compute the diffusion constant using the Einstein relation. Thermal rate constants were determined from least-squares fits to a single exponential of the temperature after switching the reference temperature at time $t = 0$ from 350 to 320 K. Each case was repeated 8 times, yielding 8 independent determinations k_i of the rate constant; we report the averages $\overline{k}$ with standard uncertainty σ computed from

$$\sigma^2 = \sum_{i=1}^{8}(k_i - \bar{k})^2/_{56}.$$

Results and discussions

Figure 2 shows that the average temperature deviates within the expected boundaries from the reference temperature, i.e., the temperature derived from the average kinetic energy converges to the reference temperature used in the algorithm. The time step was 2 fs in these simulations. Average temperature deviation is very small, within the expected limits, below 0.25 K.

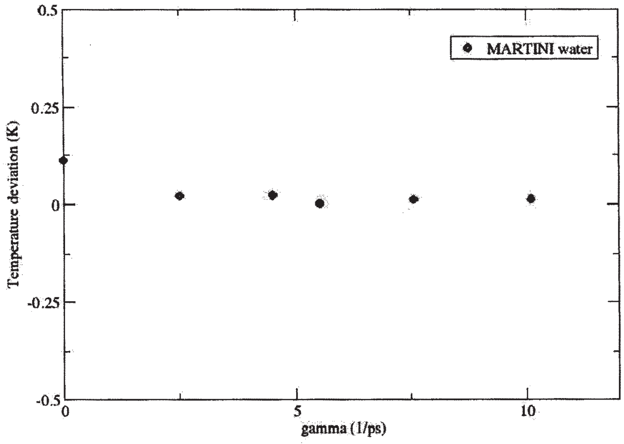

Fig. 2. Deviation from the reference temperature as a function of friction rate γ for MARTINI water. The average temperature deviation is very small, within the expected limits, below 0.25 K

For prediction of the thermal rate the following properties of MARTINI water at 320 K were used:

$$c_V = 0.0234(3)\,\text{kJ mol}^{-1}\ \text{K}^{-1},$$

$$\left\langle 1 - r/_{r_c}\right\rangle = 0.24336, \left\langle (1 - r/_{r_c})^2\right\rangle = 0.089437$$

For the Langevin case, the equation for the thermal rate constant for a fluid is given by:

$$k_{thermal} = \frac{0.5nk_B}{C_V\tau_T} + \frac{3k_B}{2c_V}\frac{[1 - exp(-2\gamma\Delta t)]}{\Delta t} \qquad (29)$$

where: $\gamma = -[ln(1 - f)]/_{\Delta t}$.

For higher values of the time constant τ_T of the global Berendsen thermostat (τ_T = 0.1 in figure 3 the equation for the thermal rate constant $k_{thermal}$ approaches that for an ideal gas). We note that the rate equation is valid when the rate constant of the thermostat is smaller than the rate of exchange between kinetic and potential energy of the system, which gives the system time to equilibrate between kinetic and potential degrees of freedom. Usually this condition is fulfilled. If, on the other hand, k_{th} is much larger (the case of a 'fast' thermostat) c_V in (14) should be replaced by its ideal-gas value $3k_B/2$.

Figure 3 compares the theoretical prediction with the simulated thermal rate constants. The agreement between theoretical prediction and simulation values is quite good. We measured the self-diffusion as an indicator of the preservation of the the dynamic properties of the system.

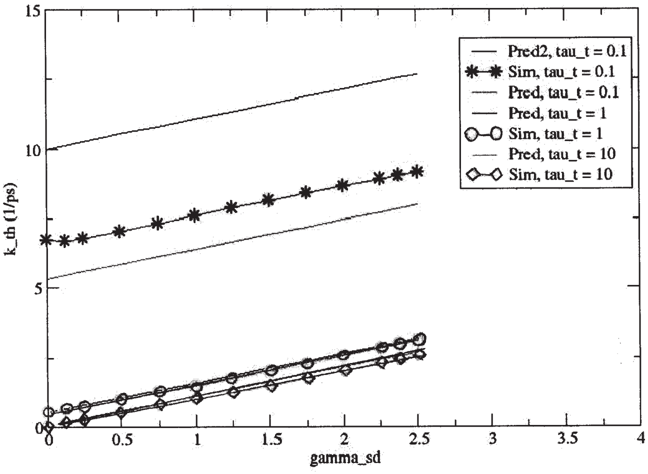

Fig. 3. Thermal relaxation rate versus friction rate for MARTINI coarse grained water, simulated and calculated with impulsive Langevin algorithm for τ_T = 0.1, 1, 10. For τ_T = 0.1

The inverse diffusion constants are given in figure 4 for the mixed thermostat. As expected, the global Berendsen thermostat did not influence the diffusion, the data points being almost identical for three different values of the coupling parameter τ_T.

However, and also as expected, the coupling strength of the stochastic thermostat heavily influences the diffusion. A low coupling strength for the stochastic thermostat is, therefore, required if one wishes to use the global combined thermostat while keeping the dynamic properties under study. The most appropriate values for γ lie between 5 and 10 ps^{-1}.

While diffusion is not influenced by the coupling strength of the global Berendsen thermostat, it does heavily depend on the strength of the stochastic thermostat coupling.

Conclusions

In this article we have shown that the combination of the global Berendsen thermostat and the stochastic thermostat provides a predictable thermostat behavior with temperatures that do not deviate from the reference and with measured thermal relaxation rates that agree with the theoretical prediction. The method described here for the combination of the two thermostats (global Berendsen and stochastic thermostat) has proved to be reliable and easy to implement. It provides a stable algorithm that allows smooth changes to new values of temperature without the need for intermediate adjustments. Further work includes the application of this mixed thermostat to multiscaling MD.

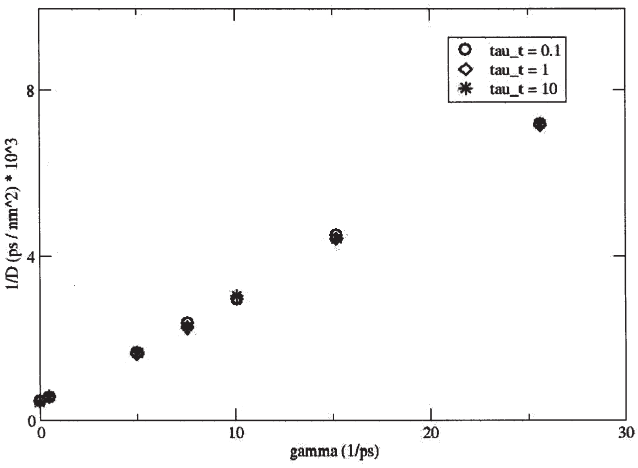

Fig. 4: Inverse diffusion coefficients versus friction rate for MARTINI coarse grained water, simulated with combined Berendsen - impulsive Langevin algorithm.

As future work, we plan to apply the presented techniques for studying the properties of polymers with the scope of designing new materials based on polymers.

References

1. HIRVI, J.T., PAKKANEN, T.A. – The Journal of Chemical Physics, **125**, 2006, p. 144712 – 1

2. YERMOLENKO, O.A., KORNICH, G.V., BERTZ, G. - Bulletin of the Russian Academy of Sciences: Physics, **72**, 2008, p. 579

3. REITH, D., MILCHEV, A., VIRNAU, P., BINDER, K. – Macromolecules, ACS Publications, **45**(10), 2012, p. 4381

4. HARMANDARIS, V.A., MAVRANTZAS, V.G., THEODOROU, D.N. - Macromolecules, ACS Publications, 1998, **31**, p. 7934

5. GOGA, N., RZEPIELA, A., MELO, M. N. , DE VRIES, A.H., HADAR, A., MARKVOORT, A.J., NEDEA, S., BERENDSEN, H.J.C. - Advances in Planar Lipid Bilayers and Liposomes, **15**, 2012 Elsevier.

6. CHRISTEN, M., VAN GUNSTEREN, W.F. - J. Chem. Phys, **124**, 2006, p. 154106

7. BERENDSEN, H.J.C., POSTMA, J.P.M., VAN GUNSTEREN, W.F., DINOLA, A. and HAAK, J.R., - J. Chem. Phys., **81**, 1984, p. 3684

8. BERENDSEN, H.J.C., - Computer Simulations in Material Science, 1991, p. 139

9. GOGA, N., RZEPIELA, A.J., DE VRIES, A.H., MARRINK, S.J., BERENDSEN, H.J.C. -. J. Chem. Th. Comp., in press, 2012

10. HESS, B., KUTZNER,C., VAN DER SPOEL, D. and LINDAHL, E., - J. Chem. Theory Comput., **4**, 2008, p. 435

11. MARRINK, S.J., RISSELADA, H.J., YEFIMOV, S., TIELEMAN, D.P. and De VRIES, A.H., - J. Phys. Chem. B, **27**, 2007, p. 7812

12. VAN DER SPOEL, D., LINDAHL, E., HESS, B., VAN BUUREN, A.R., APOL, E., MEULENHOFF, P.J., TIELEMAN, D.P., SIJBERS, A.L.T.M., FEENSTRA, K.A., VAN DRUNEN, R. and BERENDSEN, H.J.C., - Gromacs User Manual version 4.5.5., 2011, www.gromacs.org.

Manuscript received: 14.03.2012